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Lift, drag and thrust at high flight Mach number

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To assess the aerodynamic merits of a particular configuration it is convenient if similarity parameters can be found that allow comparison with other vehicles at different Mach numbers and lift coefficients. A pair of such similarity parameters occurs naturally in the study of 'direct' and 'interference' lift. The use of these parameters assists in the optimization of lift per unit drag at supersonic speeds. The designer can, for example, use these improvements to reduce wing size and structure mass. This paper derives these similarity parameters and illustrates their use.

Keywords: hypersonic aerodynamics; orbiter performance; hypersonic similarity parameters; interference lift; external burning; lifting re-entry

1. Similarity parameters for the assessment of lift and drag for lifting vehicles at high flight Mach number

'Direct' pressure is (as defined by Roe (1964)) the pressure acting on a surface due to the inclination of that surface to the flow, and a number of aerodynamic theories (e.g. tangent-wedge theory and Newtonian theory) have sought to estimate this pressure. It is most easily illustrated by the two-dimensional wedge shown in figure 1, where the pressure coefficient on the wedge can be found (NACA 1953) in terms of the free-stream Mach number (M_∞) and either shock strength ($M_\mathrm{N} = M_\infty \sin\zeta$) or the wedge angle ($\delta$). Since the pressure coefficient here is also the lift coefficient, and the pressure-drag coefficient is $C_\mathrm{P}\tan\delta$, we can write

$$\frac{C_\mathrm{L}}{C_\mathrm{D}} = \frac{C_\mathrm{L}}{C_\mathrm{DW} + C_\mathrm{DF} + C_\mathrm{DB}} = \frac{C_\mathrm{L}/C_\mathrm{DW}}{1 + (C_\mathrm{DF}/C_\mathrm{DW}) + (C_\mathrm{DB}/C_\mathrm{DW})}, \tag{1.1 a}$$

in which

$$\frac{C_\mathrm{L}}{C_\mathrm{DW}} = \cot\delta = \frac{2 - C_\mathrm{L}}{C_\mathrm{L}}\left(\frac{4 + (\gamma+1)C_\mathrm{L}M_\infty^2}{4(M_\infty^2 - 1) - (\gamma+1)C_\mathrm{L}M_\infty^2}\right)^{1/2}, \tag{1.1 b}$$

and the inviscid lift per unit drag is clearly a function of free-stream Mach number and lift coefficient. More generally, the lift-to-drag ratio of a lifting vehicle depends, in practice, on the free-stream Mach number, the skin friction drag, and the lift coefficient of the vehicle.

This relationship is applicable to a range of vehicle planforms that support a plane shock wave, and, more generally, is found to be representative of a wide range of vehicles where the lift-to-drag ratio is important (figure 2). In general, it is not too difficult to design vehicles that approach the lift-to-drag ratio of equation (1.1 b) or even to exceed it slightly. However, to obtain a significant improvement requires an

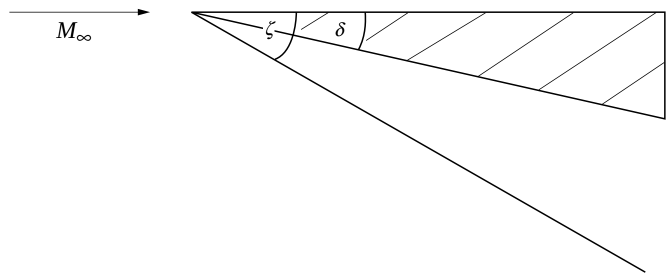

Figure 1. Plane wedge with a shock wave.

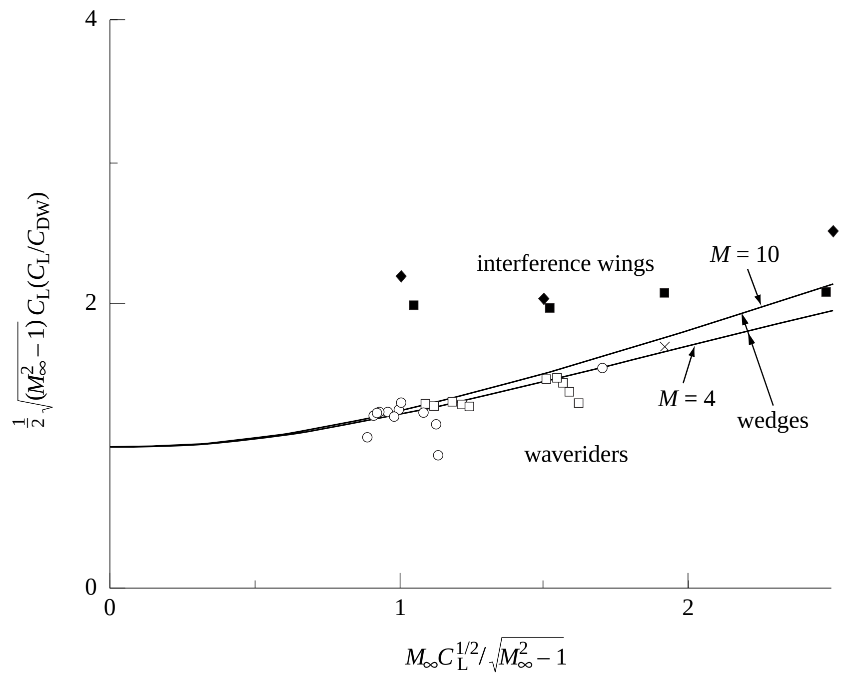

Figure 2. Performance of wings and wedges (including experimental data due to Corda & Anderson (1998) and Pike (1970*b*, 1990)).

understanding of the arrangement of aerodynamic surfaces to incorporate 'interference' lift without a more than compensating loss of direct lift. This process illustrates the similarity parameters required.

For the two-dimensional wedge of figure 1, static pressure is not only increased on the surface of the wedge, but throughout the flow between the surface and the shock wave. Thus, if the flow below the wedge is contained on one side by a vertical side plate parallel to the flow, the pressure in the flow will give a side force on that plate even though it is at zero incidence to the flow (a fact that is evident in the design of many supersonic intakes). This pressure is termed an 'interference' pressure, and the force it produces an interference side force. If this force can be contrived to be normal to the flightpath, and in the vertical plane, it becomes an interference lift.

The simplest form of interference wing is obtained (Roe 1964) by using an unswept wedge with an endplate on its side to form one-half of a lifting wing. When aerodynamically unnecessary parts of the wedge are removed, Roe (1964) finds that for small deflection angles, the inviscid lift-to-drag ratio is given by

$$\tfrac{1}{2}\sqrt{(M_\infty^2-1)}C_{\mathrm{L}}(C_{\mathrm{L}}/C_{\mathrm{DW}}) = \ln(4\sqrt{(M_\infty^2-1)/(\gamma+1)M_\infty^2 C_{\mathrm{L}}}). \tag{1.2}$$

In a more general study of wings having both direct and interference lift, Pike (1970*a*) shows that the maximum inviscid lift-to-drag ratio for such wings with small wedge angles is given by

$$\tfrac{1}{2}\sqrt{(M_\infty^2-1)}C_{\mathrm{L}}(C_{\mathrm{L}}/C_{\mathrm{DW}}) = 2\sqrt{(M_\infty^2-1)/(\gamma+1)C_{\mathrm{L}}M_\infty^4}, \tag{1.3}$$

and the wings can have all leading edges swept.

Equations (1.2) and (1.3) can be compared with the linear theory equivalent of equation (1.1), that is

$$\tfrac{1}{2}\sqrt{(M_\infty^2-1)}C_{\mathrm{L}}(C_{\mathrm{L}}/C_{\mathrm{DW}}) = 1,$$

but it is more applicable to use the (second-order accurate) direct lift expression of equation (1.1) for small deflection angles, that is

$$\tfrac{1}{2}\sqrt{(M_\infty^2-1)}C_{\mathrm{L}}(C_{\mathrm{L}}/C_{\mathrm{DW}}) = \left(1 - C_{\mathrm{L}} + \tfrac{1}{4}(\gamma+1)\frac{C_{\mathrm{L}}M_\infty^4}{M_\infty^2-1}\right)^{1/2}. \tag{1.4}$$

It is clear from inspection of equations (1.2)–(1.4) that, for small angles of attack, the wings may be compared on a single plot using the similarity parameters:

$$\tfrac{1}{2}\sqrt{(M_\infty^2-1)}C_{\mathrm{L}}(C_{\mathrm{L}}/C_{\mathrm{DW}}) \quad \text{and} \quad M_\infty^2 C_{\mathrm{L}}^{1/2}/\sqrt{M_\infty^2-1}.$$

We see from figure 3 that the interference lift vehicles of Roe (1964) can achieve a higher lift coefficient at given wave drag coefficient (or higher L/D_{W} at given C_{L}) for values of $M_\infty^2 C_{\mathrm{L}}^{1/2}/\sqrt{M_\infty^2-1}$ below about 0.8; by comparison, vehicles having both direct and interference lift (Pike 1970*a*) can have significantly better lift-to-drag ratios.

Of course, it is not satisfactory for the similarity parameters to be confined to small flow deflections. However, due to a fortuitous cancellation of errors, they are applicable up to much larger lift coefficients than is to be expected from equations (1.2)–(1.4). It has also been shown by Roe (1964) and Pike (1970*a*) that the same similarity parameters occur in the hypersonic limit. In figure 4, direct and optimum interference wings are compared using the same similarity parameters with results for Mach numbers 2, 4 and 10 derived directly from equation (1.1 *b*). The performance improvement of the optimum interference wing is confirmed, but with some variation for the particular Mach number.

In figure 5, the aerodynamically optimum interference wing is compared with the two-dimensional wedge using the parameter $M_\infty^2 C_{\mathrm{L}}^{1/2}/\sqrt{M_\infty^2-1}$; the results from Mach 2 to Mach 10 lie almost on top of one another (and tend towards the linear theory result for small values of this parameter, and towards the hypersonic small disturbance theory for large values). The increase in the lift-to-drag ratio above that of the wedge is shown, in figure 5, to correlate well when plotted against $M_\infty^2 C_{\mathrm{L}}^{1/2}/\sqrt{M_\infty^2-1}$ for Mach numbers from 2 to 10.

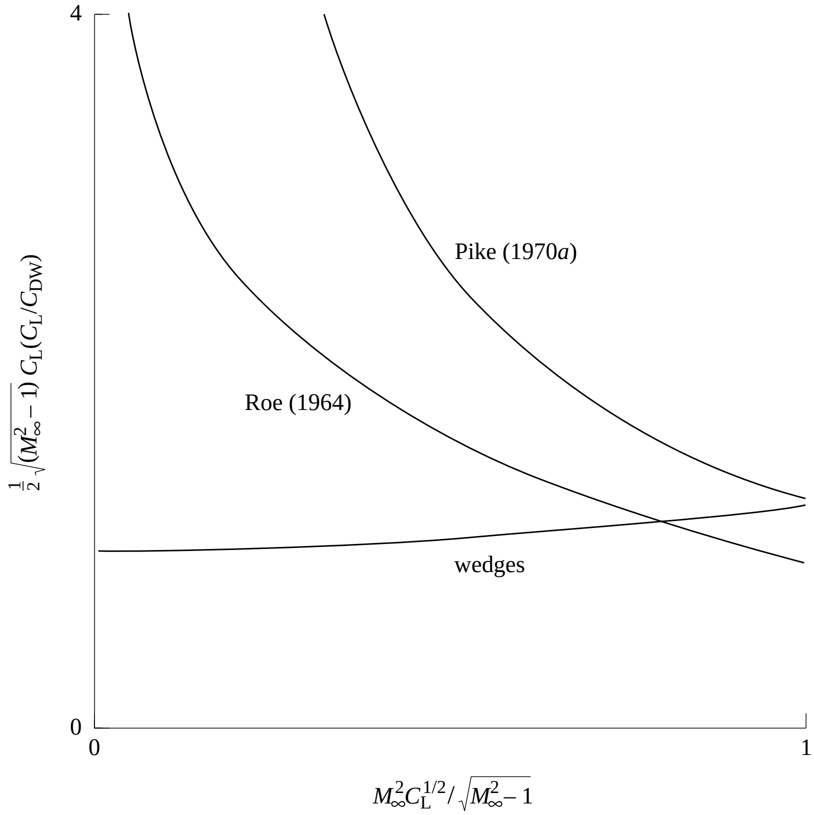

Figure 3. Wedges and interference wings at small incidence.

2. Wings with external heat addition

A further example of wing design is provided by external heat addition. As proposed by Oswatitsch (1959), the addition of heat in the flow beneath a supersonic wing can give a thrust force or drag reduction in spite of the fact that no conventionally ducted engine is utilized. The pressure field produced externally will also give a force that can preserve or enhance the lift already produced. Over the years, external heat addition has usually been achieved by combustion, and when regarded as a propulsive technique, it has then been criticized for producing only low levels of specific thrust (i.e. thrust per unit mass of heated flow) and a poor thrust-specific impulse. If regarded instead as a technique for reducing the drag of a lifting wing or body, external combustion (as assessed by Broadbent (see Townend 1991; Townend, this issue, pp. 2325–2331)) gives large improvements in L/D at suitably chosen C_L (as shown in figure 6). Furthermore, if comparison is made between two wings (a well-designed conventional wing and a wing of the same plan area and C_L but with external heat addition), then external heat addition not only enhances the L/D at given C_L, but does so at thrust-specific impulse values that are comparable with a conventional scramjet at the same flight Mach number. This prompts the question: is it more efficient to burn the fuel externally than to use it as an additive in the scramjet?

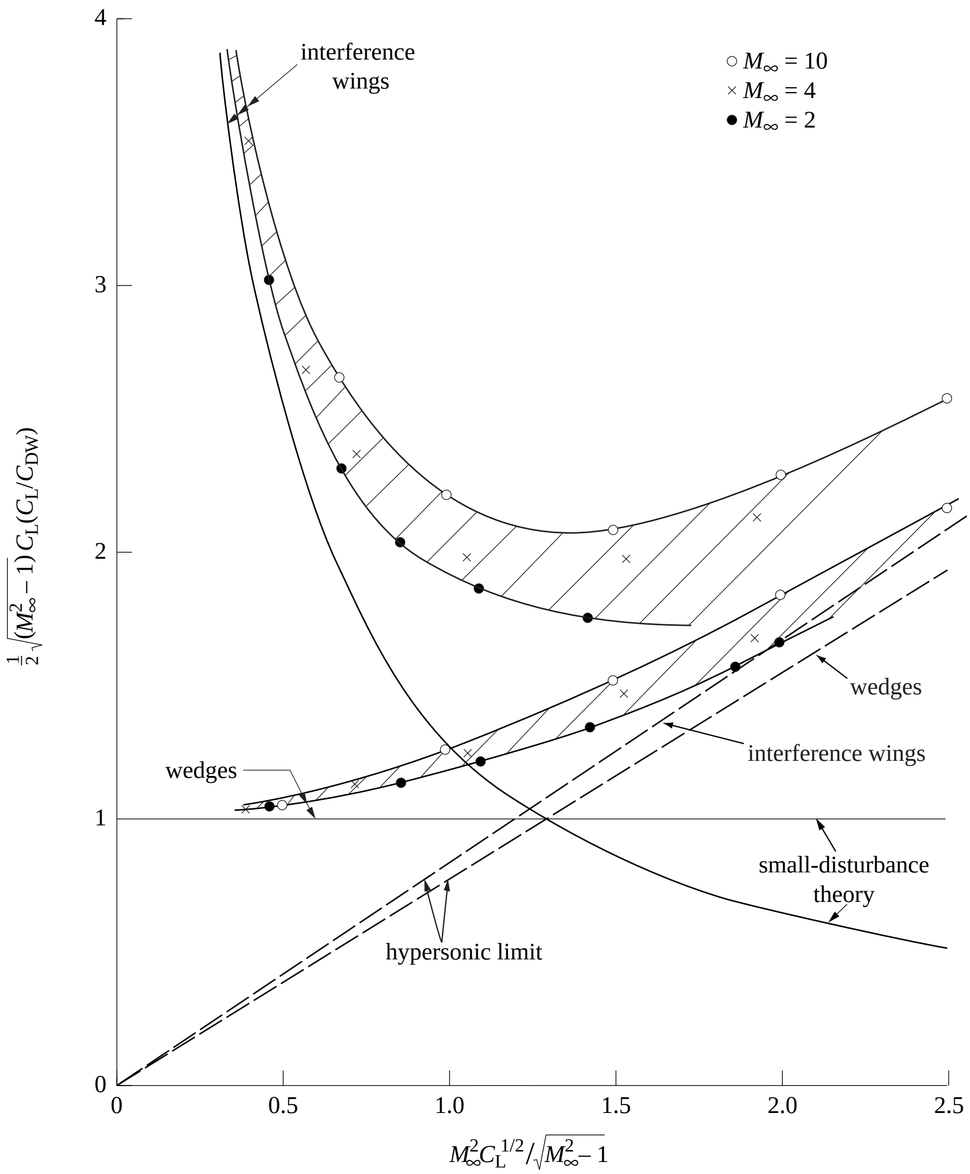

Figure 4. Comparison of the performance of interference wings with wedges.

Whatever the answer, as far as applications are concerned, external heat addition can extend the acceleration of a scramjet vehicle to higher flight Mach numbers (since the thrust minus drag of a vehicle can be improved as effectively by reducing drag as by boosting thrust). Alternatively, external heat addition could be used to enhance the crossrange of an otherwise conventional lifting re-entry vehicle, for example, by being switched in at flight Mach number 10 or so, and after re-entry itself is complete. In neither case is external heat addition required to produce a net thrust, but simply a reduction in drag.

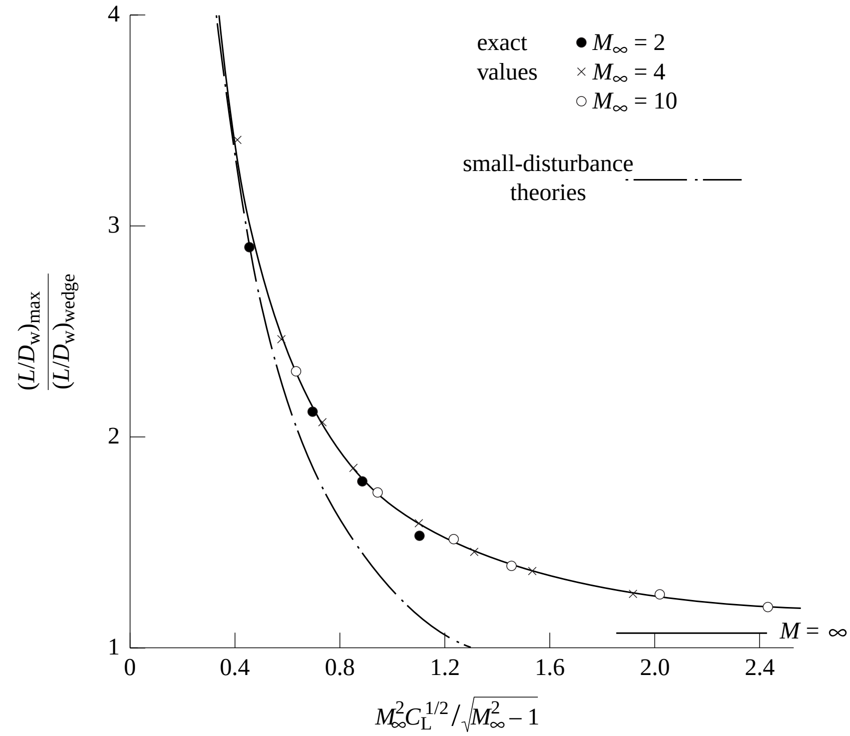

Figure 5. Ratio of L/D of interference wings to those of wedges.

3. Lifting re-entry

The refinements involved in aerodynamically optimizing a wing design are less rewarding for lifting re-entry since both C_L and M_∞ are high. The similarity parameters are still very useful, however, because they can correlate the aerodynamic performance of different vehicles over a wide range of flight Mach number. Figure 7 shows data obtained in the next two papers by Nonweiler (SLEEC22, this issue) and East (reusable space-rescue vehicle (RSRV), this issue) for glide vehicles returning relatively high crossranges (of the order of 3000 and 2000 km, respectively). Since SLEEC22 has a lower wing loading and a higher L/D, it is more responsive to skin friction drag, but it presents, nonetheless, a different curve between Mach numbers 20 and 10; as already seen (see p. 2140), the RSRV operates on almost the same curve as did the Space Shuttle on flight number STS-5.

4. Conclusions

Similarity parameters are derived and illustrated by examples of wing design both with and without the complication of external heat addition. In both cases, they assist aerodynamic optimization.

In general practice, the vehicle designer's choice will be based on more than aerodynamic merit. The aerodynamically optimum planform, dihedral, anhedral and sweep angles will not necessarily survive the scrutiny of materials experts or of the stress office, and configuration design must accommodate the requirements of propulsion,

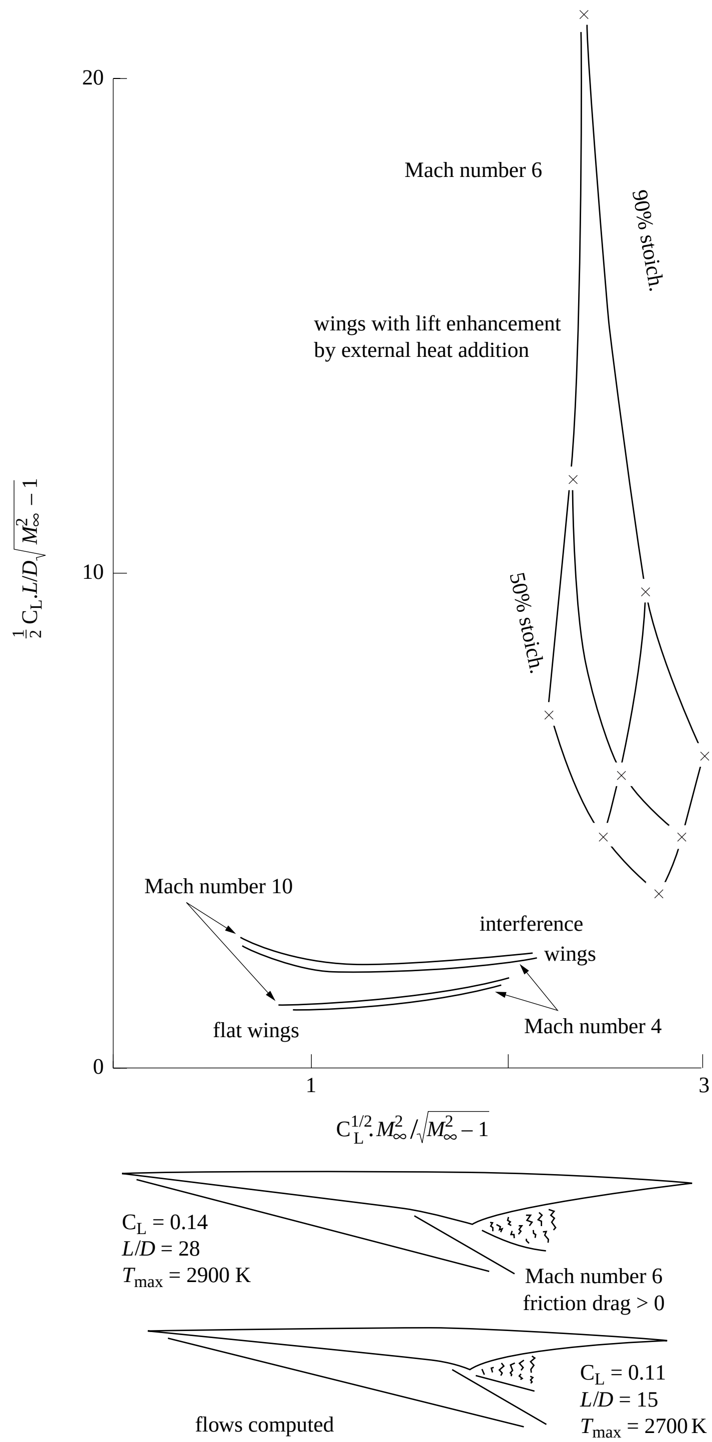

Figure 6. Wedge flows, wings and external heat addition (data due to Pike and Broadbent).

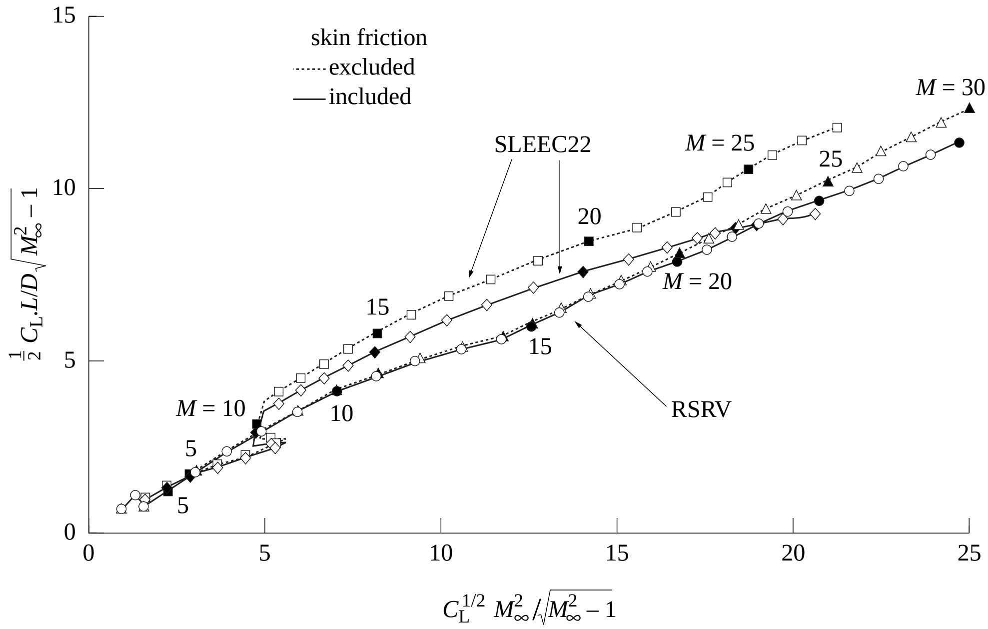

Figure 7. Performance in lifting re-entry.

volumetric efficiency and considerations such as guidance and control. At the aerodynamic level, the analysis presented is flexible and has been found useful both in selecting particular wings at a given design Mach number, and in correlating the aerodynamic performance of vehicles having no one design Mach number, such as SLEEC and the RSRV (see Nonweiler, this issue; East, this issue; Topic I).

Nomenclature

C_D	drag coefficient
C_{DB}	coefficient of base drag
C_{DF}	coefficient of friction drag
C_{DW}	coefficient of wave drag
C_L	lift coefficient
L/D	lift-to-drag ratio
M_N	Mach number component normal to a shock wave (see figure 1)
M_∞	free-stream Mach number (see figure 1)
δ	flow deflection (see figure 1)
ζ	shock wave angle (see figure 1)

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